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WAVELET ENTROPY-BASED DAMAGE IDENTIFICATION TECHNIQUE FOR HYBRID FRP-CONCRETE STRUCTURES

Mohammad, M.M. Moravvej
University of Calgary, Canada

Mamdouh El-Badry
University of Calgary, Canada

Parham Joulani
University of Calgary, Canada

ABSTRACT

Civil infrastructure, particularly bridges, is designed and built to be safe against failure and to perform satisfactorily during their service life. However, this infrastructure has been deteriorating at an alarming rate due to aging, inadequate maintenance, adverse environmental conditions, and constantly growing transportation demand. Utilization of corrosion-resistant composite materials, particularly fibre-reinforced polymers (FRP), has been an encouraging solution for the durability problems of concrete bridges. However, similar to conventional structures, the performance of hybrid FRP-concrete structures can be affected by various types of damage. Therefore, in order to ensure both safety and serviceability of these structures, it is essential to identify damage at the earliest possible time. In this paper, a damage identification technique (DIT) capable of detecting and localizing structural damage, and estimating its severity is presented. The DIT is based on the following: (1) structural damage changes the energy distribution of the acceleration signals of structural components under ambient vibrations; (2) these changes are detectable by means of discrete wavelet transforms (DWTs); and (3) the detected changes can be quantified using spectral entropy. The efficiency of the DIT is illustrated experimentally on a hybrid FRP-concrete bridge truss girder tested under static loading up to failure. The truss girder consists of pretensioned top and bottom concrete chords connected by precast web elements made of glass-FRP (GFRP) tubes filled with concrete. The results have demonstrated that the wavelet entropy-based DIT is able to detect damage in hybrid structural elements and is capable of localizing the damage and estimating its severity.

Keywords: Bridges; damage identification; FRP; hybrid structures; vibration; wavelet entropy

1. INTRODUCTION

Over the past few decades, concrete bridges in many parts of the world have been deteriorating dramatically due to aging, inadequate maintenance, excessive loading, economically driven design and construction practices, and adverse environmental conditions. Corrosion of reinforcing steel is a major source of deterioration of concrete bridges. Concrete cracking reduces the structural stiffness and expedites corrosion of the steel reinforcement. Recently, fibre-reinforced polymers (FRPs) have been utilized in concrete structures replacing steel to mitigate the durability problems of concrete bridges and to enhance their structural performance (El-Badry 2007). Hybrid FRP-concrete bridges are promising systems for developing sustainable transportation infrastructure and monitoring their structural conditions is becoming significantly important. Therefore, robust damage identification techniques (DITs) are needed to enhance public safety and to mitigate economic losses through evaluation of the structural conditions of bridge infrastructure.

Various DITs can be generally classified as (Carden and Fanning, 2004): (1) detection of presence of damage; (2) determination of location of damage; and (3) quantification of severity of damage. The outcome of a successful DIT can be used for prediction of the remaining service life of structures. From another point of view, DITs can be classified into either local or global techniques (Doebling et al. 1996). Despite all the advances in bridge monitoring techniques, the following problems have been historically difficult to solve: (1) in most in-situ cases, data from the intact (undamaged) state of in-service bridges are not available; therefore, it is not possible to simply compare before and after damage states to evaluate the current condition of bridges and their elements (Fan and Qiao 2010); (2) measuring input excitations of bridges for evaluating their global dynamic properties is not practical since it requires the bridges' normal operations be interrupted (Farrar et al. 1999); (3) local methods provide only a localized knowledge of the structure's condition and require the vicinity of damage to be known in advance and be accessible for testing (Doebling et al. 1996); (4) most of the current DITs are designed for specific types of structure and are limited to identifying a particular type of damage (Carden and Fanning 2004); and (5) bridges experience varying operational and environmental conditions which lead to changes in measured dynamic responses; these changes can be wrongly interpreted as an indication of damage (Farrar and Worden 2012).

To overcome the difficulties associated with the traditional techniques, a wavelet entropy-based DIT capable of detecting and localizing structural damage and estimating its severity is presented in this paper. The proposed DIT combines discrete wavelet transforms (DWTs) and spectral entropy for detecting and quantifying the damage-induced disturbances in the measured acceleration signals, as shown in Figure 1. The main advantages of the presented technique compared to traditional ones are: (1) it is a reference-free technique, i.e., there is no need to obtain the vibrational data of the undamaged state of structures; (2) it is a response-only technique, i.e., there is no need to control or even measure the input excitations; (3) it is capable of evaluating both the global dynamic properties of structures and the local structural condition of their elements; and (4) it can be utilized in identification of different types of damage in different types of structure (Moravvej et al. 2016). These advantages make the technique perfectly suitable for damage identification in hybrid FRP-concrete bridges during their service under ambient vibrations.

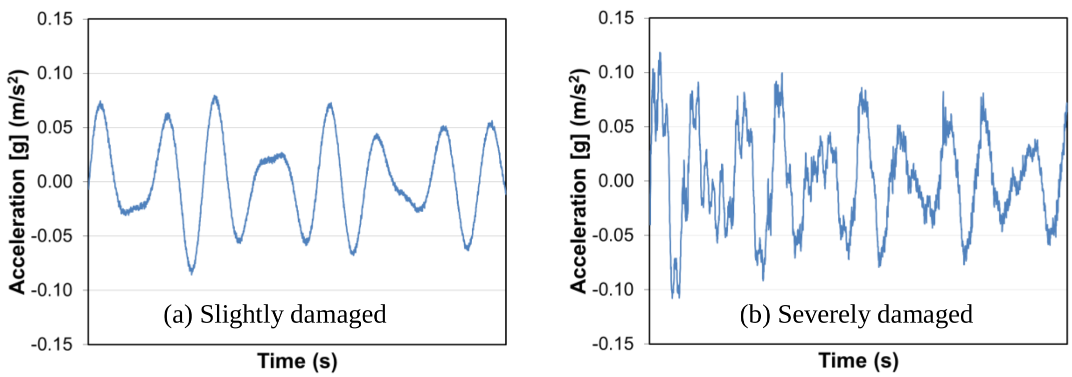


Figure 1: Acceleration signals obtained at (a) a slightly damaged location and (b) a severely damaged location

The efficacy of the proposed technique in the identification of different types of damage is examined experimentally by testing a hybrid FRP-concrete bridge truss girder system. This hybrid truss system consists of pretensioned top and bottom concrete chords connected by vertical and diagonal precast truss elements made of GFRP tubes filled with concrete. The truss elements are also reinforced with and connected to the chords by means of double-headed steel or GFRP bars. A variety of damage scenarios including failure of the truss connections and breakage of the heads of the headed bars, as well as rupture of the FRP tubes are investigated. The acceleration signals obtained at particular locations are analyzed through WT and the energy distribution of the signals is calculated. Comparing the degree of dissimilarity between the energy distributions of the signals will result in identification of possible damage.

In the following sections, the theoretical background of the technique is explained first. Then, the experimental program, including description of test specimens, instrumentation, and test setup and procedure, is described in detail. The experimental results will be presented and discussed along with the outcomes of the DIT. The results demonstrate the ability of the DIT to detect, localize, and estimate severity of the damage in the elements tested.

2. THEORITICAL BACKGROUND

The following subsections provide the mathematical definitions of the wavelet transforms, wavelet energy, wavelet entropy, and relative wavelet entropy. It is also explained how these quantities can provide useful information of the signals in a simple way and how damage can be identified through proper use of these quantities.

2.1 Wavelet Transform (WT)

In general, wavelet transform (WT) is a convertor of a signal into different mathematical forms in order to disclose the hidden characteristics of the original signal and to emphasize on its specific properties that are of interest (Gao and Yan 2011). Continuous wavelet transform (CWT) is defined as the product of a continuous signal, $f(t)$, and a basic wavelet function, $\psi(t)$. The result of this product is wavelet coefficients, defined by Eq. [1], which show how well a wavelet function correlates with the signal.

$$[1] \quad C(s,\tau) = \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} f(t) \psi^* \left(\frac{t-\tau}{s} \right) dt$$

where $\psi^*(t)$ is the complex conjugate of the wavelet, which is shifted and scaled by factors τ and s, respectively. In practice, an acceleration signal is sampled at discrete time intervals through a data acquisition system. By adopting the values of 2^j and $2^j k$ for the scale, s, and shifting factors, τ, respectively, the corresponding wavelet family can be expressed by Eq. [2] and the corresponding wavelet coefficients, $C_j(k)$, can be obtained accordingly from Eq. [1].

$$[2] \quad \psi_{j,k}(t) = 2^{-\frac{j}{2}} \psi\left(2^{-j} t - k\right)$$

Discrete wavelet transform (DWT) works as a pair of filters, which decompose the acceleration signal into low- and high-frequency components and find corresponding wavelet coefficients for each component. The low-frequency component is filtered one more time. The process repeats until the final level of decomposition, where the original acceleration signal is decomposed into j groups of wavelet coefficients, from the lowest frequency component to the highest frequency component.

2.2 Wavelet Energy

The wavelet coefficients provide full information of the signal in a simple way and can be used as a direct estimation of the wavelet energy. In this context, the energy of the signal at each scale, E_j, and the energy of the signal at each sampled time, $E(k)$, are defined, respectively, as:

$$[3] \quad E_j = \sum_k \left| C_j(k) \right|^2$$

$$[4] \quad E(k) = \sum_j \left| C_j(k) \right|^2$$

Consequently, the total wavelet energy, E_{total}, and the wavelet energy ratio, p_j, of the j^{th} scale can be obtained by:

$$[5] \quad E_{total} = \sum_j \sum_k \left| C_j(k) \right|^2 = \sum_j E_j$$

$$[6] \quad p_j = \frac{E_j}{E_{total}}$$

The wavelet energy ratio vector, $\{p_j\}$, represents the energy distribution of the signal over different frequency bandwidths and provides a suitable tool for detecting and characterizing singular features in the signal.

2.3 Wavelet Entropy (WE)

The entropy, in general, is a quantitative measure of the degree of disorder in a system (Shannon 1948). Therefore, the wavelet entropy can quantify the degree of disorder in a measured acceleration signal and is defined as (Powell and Percival 1979; Rosso et al. 2001):

$$[7] \quad S_{WT}(p) = -\sum_j p_j \ln\left[p_j\right]$$

If damage occurs at a location, the degree of disorder of the acceleration signal obtained at that location increases because of energy dissipation mechanisms and increases in nonlinearity due to gaps and frictions (Lee et al. 2014). Consequently, the probabilistic distribution of wavelet energies increases. As a result, the wavelet entropy of the signal also increases which can be utilized as an effective quantitative measure of the damage severity.

2.4 Relative Wavelet Entropy (RWE)

RWE describes the degree of dissimilarity between two sets of signals and can be defined as:

$$[8] \quad S_{WT}(p|q) = \sum_j p_j \ln\left[\frac{p_j}{q_j}\right]$$

RWE will be equal to zero only if the wavelet energy ratio vectors $\{p_j\}$ and $\{q_j\}$ are exactly the same. For the application of RWE in damage detection, these two sets of signals must be chosen in such a way that the degree of dissimilarity between them represents the severity of possible targeted damage.

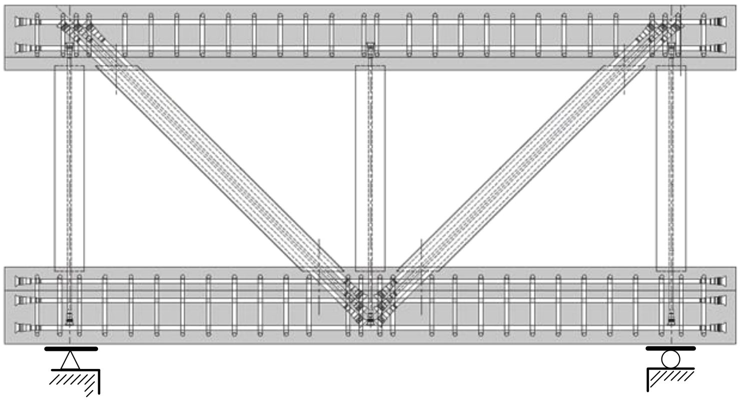
3. EXPERIMENTAL PROGRAM

The experimental program consists of fabricating bridge truss girder specimens, testing them under static loading, and performing series of impact tests on both undamaged and damaged states of the specimens. The experimental program is generally designed to be applicable to in-situ dynamic tests on in-service bridges under ambient vibrations. The truss girder specimens have originally been designed and fabricated as part of an ongoing comprehensive research program on the development of a novel corrosion-free bridge system (El-Badry 2007, Joulani et al. 2016). Each truss girder is generally made of a specific number of typical panels, positioned symmetrically on each side of the girder mid-span to accommodate the required span length of the bridge. Two full-size 2-panel truss girder specimens, Girder 1 and Girder 2, were considered in the experimental program reported herein.

3.1 The Girder Specimens

The truss girders consist of pretensioned top and bottom concrete chords connected by precast vertical and diagonal truss elements made of glass fiber-reinforced polymer (GFRP) tubes filled with concrete. The truss elements are reinforced and connected to the chords by means of double-headed GFRP bars in Girder 1 and by means of double-headed steel bars in Girder 2. The vertical truss elements are predominantly in compression and the diagonal elements are mainly in tension. The GFRP tubes enhance the compressive strength of the verticals by confining the concrete core while the double-headed bars serve as internal reinforcement with excellent anchorage properties in the diagonals. The top and bottom chords of Girder 1 are reinforced with longitudinal GFRP bars for flexural resistance and control of cracking and with GFRP stirrups to provide shear resistance. In Girder 2, steel rebars and stirrups are used to reinforce the chords. A 3-m long two-panel truss girder in the test frame is shown in Figure 2a. Typical reinforcement of the specimens is illustrated in Figure 2b.

(a) Two-panel truss girder specimen



(b) Typical reinforcement of a two-panel truss girder specimen

Figure 2: A two-panel hybrid FRP-concrete truss girder specimen and its typical reinforcement

3.2 Instrumentation

3.2.1 Dynamic Excitation

Dynamic excitations were induced in the girder specimens in two ways: (1) by applying impact forces using a hammer in order to excite a few modes of vibration of particular elements of the girder; and (2) by applying a vertical cyclic load at mid-span of the girder to simulate bridge ambient vibrations due to traffic loads. It should be noted that since the proposed DIT is a response-only technique, the impact forces were neither controlled nor recorded for the damage identification process. In fact, ambient vibrations, due to traffic or wind loads, shall be adequate for damage identification in bridges using the proposed technique.

3.2.2 Accelerometers:

The acceleration signals were measured by means of accelerometers, which were temporarily attached to the specimens at different locations according to the testing procedure.

3.2.3 Strain Gauges

EA-Series strain gauges were used in the experimental program for measuring the strain in the reinforcement, the GFRP tubes, and the headed bars. The data obtained through the strain gauges are used to verify the location and the severity of damage identified by the proposed technique.

3.2.4 Data Acquisition System

A data acquisition system and a personal computer equipped with MATLAB were utilized to obtain signals from the accelerometers and to perform signal processing on the measured signals. The data was sampled at the rate of 6 kHz.

3.3 Testing Setup and Procedure

Two types of testing were conducted on the girder specimens: (1) none destructive testing (NDT), in which the specimens and/or their structural elements were dynamically excited by means of impact forces; and (2) destructive testing (DT), in which a vertical static load was applied at mid-span and increased monotonically from zero to failure to produce various types of damage in the different elements of the girders (Joulani et al.2016).

3.3.1 Vertical Truss Elements

Accelerometers were attached to the vertical elements of Girder 1 at three locations along their heights, as shown in Figure 3, establishing a 3×3 matrix of locations, as given in Eq. [9]. The elements of each column of the matrix represent the locations of the sensors on each vertical tube. A series of impact tests using the hammer was conducted before, during, and after the static loading test using only two accelerometers at a time to cover all the nine measurement points. Also, one accelerometer was kept attached to L22 during the static loading test to record 3 seconds long acceleration signals at every 200 kN of the load.

[9] $$[\text{Location}]_{3x3} = \begin{bmatrix} \text{L11} & \text{L12} & \text{L13} \\ \text{L21} & \text{L22} & \text{L23} \\ \text{L31} & \text{L32} & \text{L33} \end{bmatrix}_{\text{Vertical}}$$

3.3.2 Truss Connections

In order to evaluate the performance of the truss connections, accelerometers were attached to the diagonal elements of Girder 2 at two locations near their connections to the top and bottom chords as shown in Figure 4, establishing a 2×2 matrix of locations, as given in Eq. [10]. The girder was statically loaded up to 1330 kN, which was equal to the actuator maximum capacity, and then was unloaded to zero (Joulani et al. 2016). A series of dynamic excitations was induced in the Girder by application of a vertical cyclic load at the mid-span for duration of 10 seconds at 0.5 Hz frequency. The amplitude of the cyclic load was 87.5 kN (the maximum wheel load of CL-W truck according to CAN/CSA-S6-06).

[10] $$[\text{Location}]_{2x2} = \begin{bmatrix} \text{L11} & \text{L12} \\ \text{L21} & \text{L22} \end{bmatrix}_{\text{Diagonal}}$$

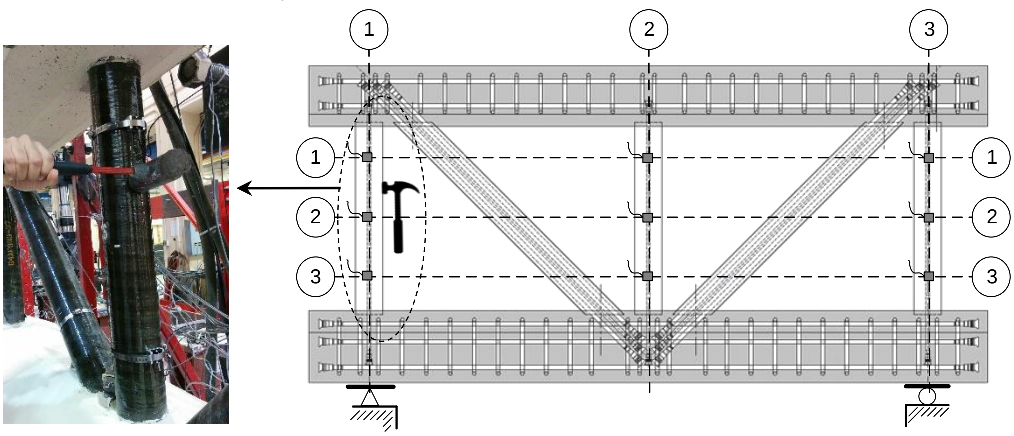


Figure 3: Typical impact tests and sensor arrangement for damage identification in the vertical truss elements

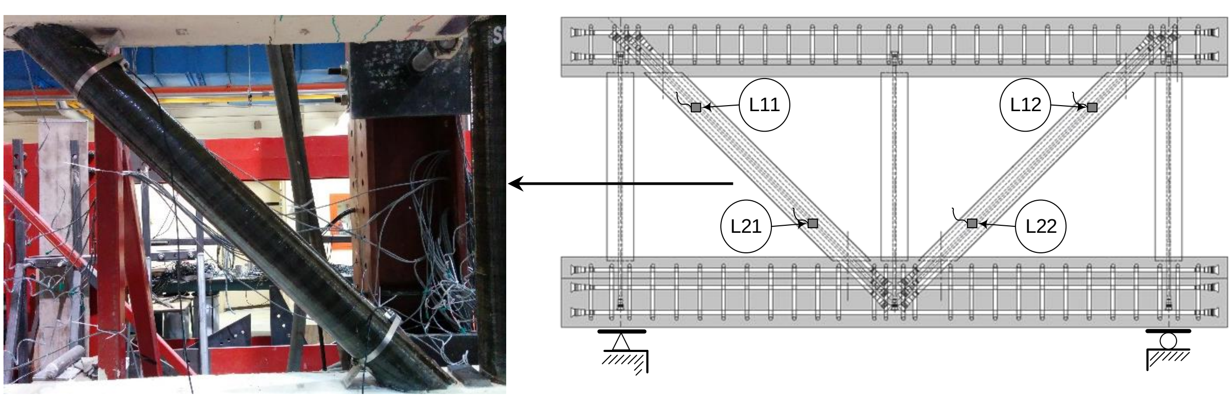


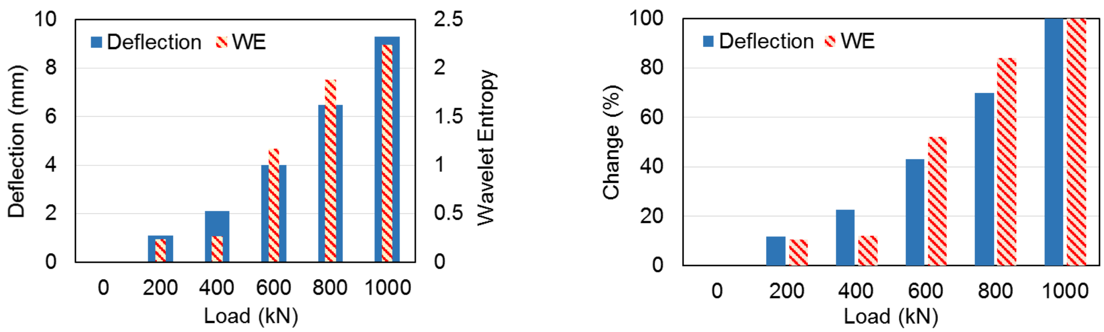
Figure 4: Typical sensor arrangement for damage identification in the truss connections

4. RESULTS AND DISCUSSION

In addition to selecting appropriate locations for measuring acceleration signals, choosing a mother wavelet that fits the pattern of damage properly results in detecting the damage content more accurately. In the present work, the measured signals have been analyzed using the Daubechies wavelet family. Among several alternatives, successful use of Daubechies wavelets in damage identification has been reported by many researchers (Ren and Sun 2008; Qiao et al. 2012; Xiang and Liang 2012; Solis et al. 2013). As explained in the experimental program, one accelerometer was kept attached at mid-height of the central vertical element its during the static loading of Girder 1. In Figure 5, wavelet entropy (WE) of the acceleration signals recorded over 3 seconds long bursts at every 200 kN is plotted along with the deflection of the girder. Both the wavelet entropy and the deflection are increasing, with relatively the same trend, with the increase in the applied load. Increase in the WE indicates an increase in the degree of disorder in the system and can be interpreted as the propagation of damage due to static loading. Formation of cracks in the concrete chords and around the connections of the truss elements to the chords, slippage of the heads of the GFRP bars inside the connections, and rupture of the GFRP tubes in compression are some of the reasons for observing the increase in the wavelet entropy of the signals obtained during static loading of Girder 1.

4.1 Vertical Truss Elements

The wavelet analysis of the acceleration signals recorded by sensors during the impact tests on the vertical truss elements results in a 3×3 matrix of wavelet energy ratio vectors, as given in Eq. [11]. Each cell in the matrix is a vector representing the energy distribution of the signals over frequency bandwidths. Comparison of any two wavelet energy ratio vectors in Eq. [11] using Eq. [8] describes the degree of dissimilarity between the two energy distributions, which are utilized here to identify possible damage.



(a) Deflection vs. load and WE vs. load for Girder 1

(b) Similarity of increase in WE and deflection

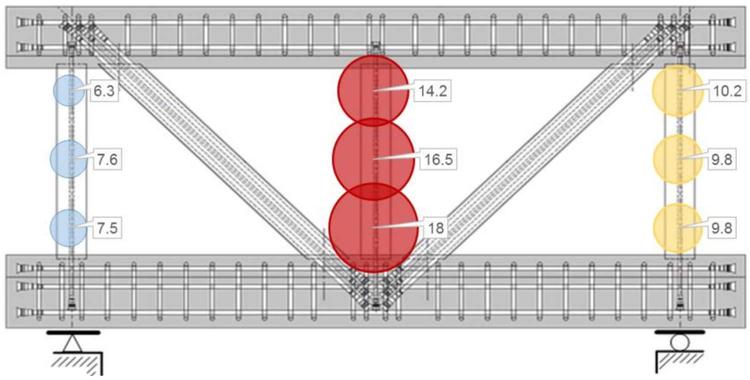
Figure 5: Increase in the wavelet entropy of the acceleration signals with the increase in the static Load

Performing the analysis for the entire matrix results in a 3×3 RWE matrix of each location relative to the eight other locations as given in Eq. [12]. This matrix identifies both the location and severity of damage. When a particular location is affected by damage, its RWE will be higher compared to others. The values in Eq. [12] are then normalized, scaled to 100, and depicted in Figure 6a. It can be seen from the figure that the central vertical element of the truss is the most critical, followed by the right element. This finding can be validated using the maximum strains in the GFRP tubes induced by the static load, as given in Eq. [13]. The results of the RWE analysis also agree with the physical damage caused by the static loading test, in which rupture of the GFRP tubes occurred only in the lower half of the central vertical truss element (see Figure 6b), while the two other GFRP tubes showed no sign of rupture.

[11] $$[P]_{3x3} = \begin{bmatrix} \{P11\} & \{P12\} & \{P13\} \\ \{P21\} & \{P22\} & \{P23\} \\ \{P31\} & \{P32\} & \{P33\} \end{bmatrix}$$

[12] $$[\mathrm{RWE}]_{3x3} = \begin{bmatrix} 22.8 & 51.4 & 36.9 \\ 27.3 & 59.5 & 35.4 \\ 27.1 & 64.8 & 35.3 \end{bmatrix}$$

$$[13]\quad [\text{Axial strain}]=\begin{bmatrix}-483 & \text{NA} & -644\\ -677 & -12282 & -1318\\ -514 & \text{NA} & -2553\end{bmatrix}\times 10^{-6} \text{ and } [\text{Hoop strain}]=\begin{bmatrix}1880 & \text{NA} & 1497\\ 281 & 6543 & 179\\ 613 & \text{NA} & 1113\end{bmatrix}\times 10^{-6}$$



(a) Normalized and scaled RWE indices

(b) Vertical truss elements after static test

Figure 6: Location and severity of damage in the vertical truss elements of Girder 1

4.2 Truss Connections

Two seconds of time-acceleration responses obtained at the truss connections during the cyclic loading on Girder 2 are shown in Figure 7. The responses at the top and bottom of each diagonal look almost identical and any possible effect of damage on the acceleration signals cannot be detected in the figure. In fact, even the natural frequency of the vibration, which was calculated using Fast Fourier Transform (FFT) analysis, is equal to 48.1 Hz for all four signals. However, when the signals are decomposed through the wavelet entropy-based DIT, the hidden characteristics of the signals are disclosed. As an example, the wavelet energy ratios of the signals obtained at L11 and L22 are compared in Figure 8a. The wavelet energy ratio of the two signals is not identically distributed over frequency bandwidths. This difference is more pronounced in Figure 8b, in which the relative wavelet energy ratio of the signals is plotted. The dissimilarity between the two energy distributions is utilized in the DIT to identify possible damage.

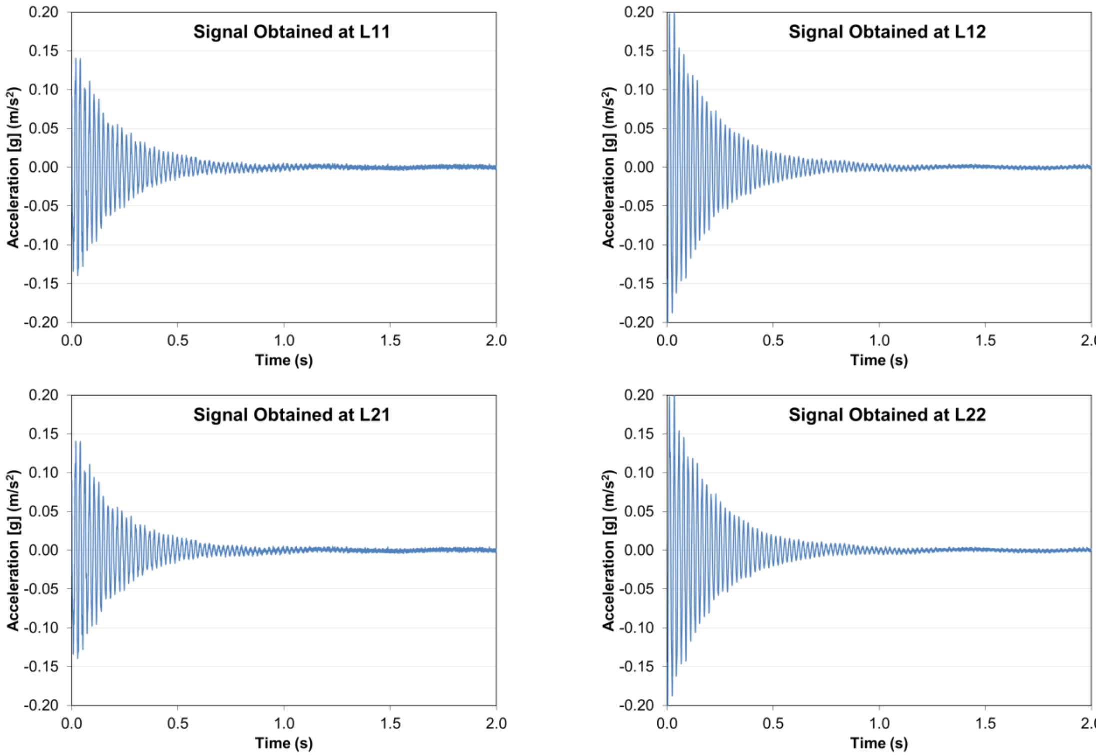
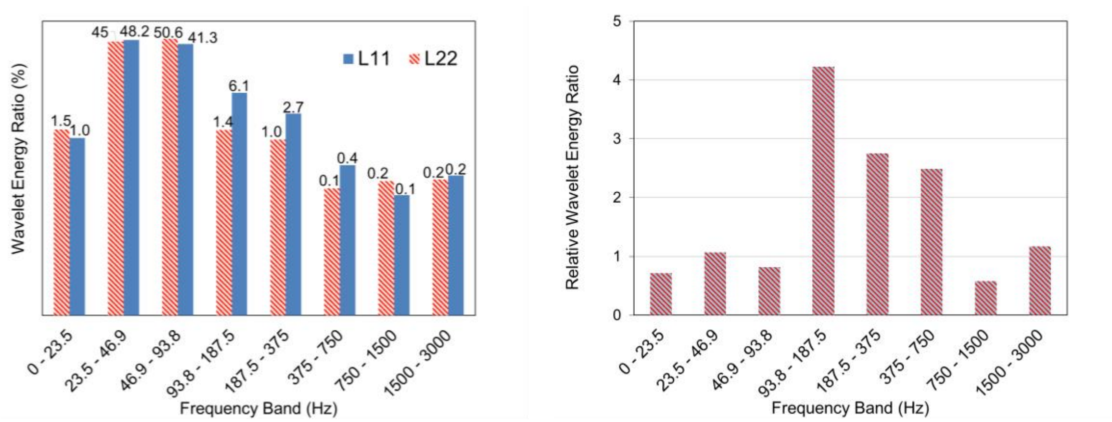


Figure 7: Acceleration signals obtained at the connections of Girder 2 under the vertical cyclic load



(a) Wavelet energy ratio of the signals

(b) Relative wavelet energy ratio of the signals

Figure 8: Comparison between the wavelet energy ratio distributions of the signals obtained at L11 and L22

The results of the damage identification in the truss connections are given as a 2×2 RWE matrix in Eq. [14]. Two main conclusions can be drawn from Eq. [14]. First, the RWE values are relatively low (e.g. in comparison to Eq. [12]), which means that the signals obtained at the truss connections are fairly behaving the same. In other words, the performance of the truss connections are not significantly affected by the static load and the connections can still perform satisfactory under the load. Second, since the RWEs at locations L12 and L22 are greater than the RWEs at locations L11 and L21, failure is more likely to occur in the connections of the right-side diagonal element (Fig. 4) before the connections of the left-side diagonal. This finding can be validated using the average plastic strains remaining in the steel headed bars in each connection after unloading the girder. The strains are given in Eq. [15]. In addition, Figure 9 shows slight cracking of concrete around the connections of the right-side diagonal element to the chords induced by the static load.

[14] $\left[\mathrm{RWE}\right]_{2x2}=\begin{bmatrix}2.0 & 3.2\\ 2.1 & 7.3\end{bmatrix}$

[15] $\left[\text{Plastic strain}\right]=\begin{bmatrix}2637 & 4788\\ 4403 & 10189\end{bmatrix}\times10^{-6}$

(a) At location L12 (b) At location L22

Figure 9: Cracking of concrete around the connections of the right-side diagonal element and the chords

5. CONCLUSIONS

Concrete bridges are crucial components of transportation infrastructure and have been in service for several decades. Nowadays many of the bridges are approaching or even exceeding their design life span. One encouraging solution for the durability problems and for enhancing the structural performance of concrete bridges is utilization of FRPs in bridge construction. Therefore, monitoring the conditions of hybrid FRP-concrete bridge infrastructure is becoming significantly important. A wavelet entropy-based damage identification technique was introduced and experimentally evaluated in this paper. The technique is both response-only and reference-free and the instrumentations are very simple. This makes the technique a practical means for damage identification in existing bridges. Utilization of the technique in structural health monitoring of bridges will enhance public safety and mitigate economic losses due to its potential to facilitate more economical maintenance and management of infrastructure. The main conclusions drawn from the present study are:

1. Hidden characteristics of acceleration-time signals obtained through the experimental program could be disclosed by wavelet transform and could be quantified by wavelet entropy.
2. The propagation of damage during the static loading test could be identified by the increase in the wavelet entropy of the acceleration signals.
3. The wavelet entropy-based damage identification technique could identify the damage content in the concrete-filled GFRP tubes of the hybrid FRP-concrete truss girder and could help in decision-making regarding maintenance of the girder. Results of the damage identification analysis were verified by the strain gauge data and visual inspection of the actual damage of the tubes.
4. The wavelet entropy-based damage identification technique could evaluate the structural condition of the connections in the hybrid FRP-concrete truss girder tested under static loading. The results were verified by the data obtained from strain gauges attached to the connection reinforcement and by visual inspection of the connections.

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